

Observer based Fault Tolerant Control for Takagi-Sugeno Nonlinear Descriptor systems

H. HAMDI[†], M. RODRIGUES[‡], Ch. MECHMECHE[†] and N. BENHADJ BRAIEK[†]

Abstract—In this paper, an observer-based Fault Tolerant control (FTC) study is proposed for nonlinear descriptor systems approximated by Takagi-Sugeno representation. A control law is designed in order to compensate the actuator faults and allows the system states to track a reference states corresponding to the original descriptor system in the fault free case. The design of such a control law requires the knowledge of the faults, for this aim, a Proportional Multi-Integral Observer (PMIO) is presented to achieve this task. The robust stability of the system with the fault tolerant control law is analyzed with Lyapunov theory. Sufficient stability conditions and the gains of the FTC are obtained in terms of linear matrix inequalities (LMIs). A numerical example is used to illustrate the efficiency of the studied method.

I. INTRODUCTION

Reliability and safety of physical process have always been a major concern for industrial manufacturers. There is an absolute necessity to identify early unexpected changes in the system before they lead to a complete breakdown. Fault Detection and Isolation (FDI) and Fault Tolerant Control (FTC) for linear and nonlinear ordinary systems have already been addressed in a large number of papers [11], [6]. FDI comprises fault detection, fault isolation and even fault identification. FTCS are generally divided into two classes: passive and active. Passive FTCS are based on robust controller design techniques and aim at synthesizing one (robust) controller that makes the closed-loop system insensitive to certain faults. This approach requires no online detection of the faults, and is therefore computationally more attractive. Its applicability, however, is very restricted due to its serious disadvantages. As opposed by the passive methods, the active approach to the design of FTCS is based on controller redesign, or selection/mixing of predesigned controllers [10]. This technique usually requires a fault detection and diagnosis (FDD) scheme that has the task to detect and localize the faults that eventually occur in the system. The FDD part uses input-output measurement from the system to detect and localize the faults. The estimated faults are subsequently passed to a reconfiguration mechanism (RM) that changes the parameters and/or the structure of the controller in order to achieve an acceptable post-fault system performance. There are a number of important issues when designing active FTCS. Probably the most significant one is the integration between the FDD part

and the FTC part. The majority of approaches in the literature are focused on systems modeled by many classes of nonlinear ordinary systems [1], [9]. Nonlinear descriptor systems are not also studied, then only a considerable amount of results have been established in the framework of linear descriptor systems. Very often the dynamics of real physical systems can not be represented accurately enough by linear dynamical models so that nonlinear models have to be used. This necessitates the development of techniques for FTCS design that can explicitly deal with nonlinearities in the mathematical representation of the system. Nonlinearities are, in fact, very often encountered in the representations of complex safety critical controlled systems. During the last two decades, fuzzy technique has been widely used in nonlinear system modeling, especially for systems with incomplete plant information. Fuzzy logic systems serve well as universal approximators [14]. The well-known Takagi-Sugeno (T-S) fuzzy model [7] is a popular and convenient tool in functional approximations. Accordingly, the stabilization problem for systems in T-S fuzzy model has been studied. Recently, a wider class of fuzzy systems described by the descriptor form is considered in [12], where the model is in the extended T-S fuzzy model. It is known that a descriptor model describes a practical system better than a standard dynamic model [4]. The descriptor system describes a wider class of systems including physical models and nondynamic constraints. In [13], a fuzzy model in the descriptor form is introduced, and stability and stabilization problems for the system are addressed.

Motivated by the above discussion, in this paper, a Proportional Multi-Integral Observer (PMIO) is developed. The proposed PMIO [2] is dedicated to the design of a fault tolerant control strategy for a class of nonlinear descriptor systems described by T-S models with measurable premise variables. This paper is organized as follows: in Section II the fuzzy T-S structure of nonlinear descriptor systems is introduced. In Section III, we study the structure and the design of the proposed Proportional Multi-Integral Observer. Fault tolerant control by state feedback is tackled in Section IV, Finally, and before concluding, an illustrative example is considered in Section V.

II. SYSTEM DESCRIPTION AND PROBLEM STATEMENT

A. Takagi-Sugeno fuzzy Model

Consider the following general form of nonlinear descriptor systems:

$$\begin{cases} E\dot{x}(t) = A(x(t))x(t) + B(x(t))u(t) \\ y(t) = C(x(t))x(t) \end{cases} \quad (1)$$

[†]H. Hamdi, Ch. Mechmeche and N. BenHadj Braiek are with Laboratory for Advanced Systems (LSA), Polytechnic school of Tunisia (e-mail: hammmdihabib@yahoo.fr, chokri.mechmeche@esst.rnu.tn, naceur.benhadj@ept.rnu.tn).

[‡]M. Rodrigues is with Laboratory Automation and Engineering Process (LAGEP), CNRS UMR 5007, University Lyon1, France (e-mail: mickael.rodrigues@univ-lyon1.fr).

where $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^p$ ($p \leq n$) and $y(t) \in \mathbf{R}^m$ represent respectively the singular state, the control input and the output vectors. $A(x(t))$, $B(x(t))$ and $C(x(t))$ are nonlinear matrices functions. For simplicity, we can always consider that $E \in \mathbf{R}^{n \times n}$ is a constant matrix, it may not have full rank.

Fuzzy descriptor system is defined by extending the T-S fuzzy ordinary model. The fuzzy T-S descriptor model is then described by the following fuzzy IF-THEN rules;

$$\text{IF } \xi_1(t) \text{ is } M_{1i} \text{ and } \dots \text{ and } \xi_p(t) \text{ is } M_{1p}, \text{ THEN}$$

$$\begin{cases} E\dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t), i = 1 \dots h \end{cases} \quad (2)$$

$A_i \in \mathbf{R}^{n \times n}$, $B_i \in \mathbf{R}^{n \times p}$, and $C_i \in \mathbf{R}^{m \times n}$ are time invariant matrices of appropriate dimensions. r is the number of IF THEN rules, and M_{ij} are the fuzzy sets. $\xi_1(t) \dots \xi_p(t)$ are premise variable. We set $\xi(t) = [\xi_1(t) \dots \xi_p(t)]$. Then the descriptor equation is defined as follows;

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^h h_i(\xi(t)) [A_i x(t) + B_i u(t)] \\ y(t) = \sum_{i=1}^h h_i(\xi(t)) C_i x(t) \end{cases} \quad (3)$$

where

$$h_i(\xi(t)) = \frac{\beta_i(\xi(t))}{\sum_{i=1}^h \beta_i(\xi(t))}, \beta_i(\xi(t)) = \prod_{j=1}^p M_{ij}(\xi_j)$$

and $M_{ij}(\xi_j)$ are the membership functions of the fuzzy sets of M_{ij} . We assume that $\beta_i(\xi(t)) \geq 0$, $i = 1 \dots h$ and $\sum_{i=1}^h \beta_i(\xi(t)) > 0$, $\forall t$ hence the weighting function $h_i(\xi(t))$ satisfy the properties of the sum convex.

$$\begin{cases} \sum_{i=1}^h h_i(\xi(t)) = 1 \\ 0 \leq h_i(\xi(t)) \leq 1 \end{cases}$$

B. Problem statement

Under actuator faults, the system (3) can be rewritten in the following form:

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^h h_i(\xi(t)) [A_i x(t) + B_i (u(t) + f(t))] \\ y(t) = \sum_{i=1}^h h_i(\xi(t)) C_i x(t) \end{cases} \quad (4)$$

where $f(t)$ is an actuator fault. It can be represented by an additive or a multiplicative external signal [9]. These malfunctions of an actuator faults can be represented by a faulty control input $u_f(t) = (I_p - \gamma)u(t)$ which can be rewritten as an external additive signal: $u(t) + f(t)$ where $f(t) = -\gamma u(t)$ with

$\gamma \triangleq \text{diag}[\gamma^1, \gamma^2, \dots, \gamma^p]$, $0 \leq \gamma^k \leq 1$ such that

$$\begin{cases} \gamma^k = 1 \rightarrow \text{a total failure of the } k^{\text{th}} \text{ actuator } k \in [1, \dots, p] \\ \gamma^k = 0 \rightarrow \text{the } k^{\text{th}} \text{ healthy actuator} \end{cases}$$

The goal of this paper is to seek a control law to ensure the closed-loop stability of the system (4) as well as the

actuator fault detection and isolation. This goal can be well accomplished by introducing the following control law:

$$u_f(t) = \sum_{i=1}^h h_i(\xi(t)) (\Gamma_i (x(t) - \hat{x}(t)) - \hat{f}(t) + u(t)) \quad (5)$$

where $\hat{x}(t)$ and $\hat{f}(t)$ are respectively the state and the fault estimations and $\Gamma_i \in \mathbf{R}^{n \times p}$ the feedback gains to be found.

Before starting the FTC design, some useful basic assumption for descriptor systems are given as follows [8] and [4]:

- **A1.** $\text{rank}(C_i B_i) = \text{rank}(B_i) = p$, $\forall i = 1, \dots, h$,
- **A2.** The triple matrix (E, A_i, C_i) is R-observable, for all $i = 1, \dots, h$, i.e.,

$$\text{rank} \begin{bmatrix} sE - A_i \\ C_i \end{bmatrix} = n, \forall s \in \mathcal{C}. \quad (6)$$

where \mathcal{C} denotes the complex plane.

- **A3.** The triple matrix (E, A_i, C_i) is Impulse-observable, for all $i = 1, \dots, h$, i.e.,

$$\text{rank} \begin{bmatrix} E & A_i \\ 0 & E \\ 0 & C_i \end{bmatrix} = n + \text{rank}(E) \quad (7)$$

- **A4.** The fault $f(t)$ is assumed to be a bounded time varying signal with null s^{th} derivative i.e. $\|f(t)\| \leq \alpha_1$ and $\|\dot{f}(t)\| \leq \alpha_2$ and $0 \leq \alpha_1, \alpha_2 < \infty$.

- **A5.** Only partial actuator faults are considered, i.e., $\gamma^k \in [0, 1]$.

In the following section, we propose to design a PMIO to estimate the system state vector and the fault signal simultaneously for T-S fuzzy descriptor model. After an efficient fault tolerant control scheme by using the estimated states and faults is developed.

III. T-S FUZZY PMIO DESIGN

By using the same idea in T-S fuzzy descriptor model, a fuzzy Proportional Multi-Integral Observer uses a number of local linear time-invariant observers. Each local observer is associated with each fuzzy rule given below:

Rule i IF $\xi_i(t)$ is M_{1i} Then

$$\begin{cases} \dot{z}(t) = N_i z(t) + G_i u(t) + L_i y(t) + H_i \hat{f}_s(t) \\ \hat{x}(t) = z(t) + M_2 y(t) \\ \dot{\hat{f}}_s(t) = \Phi_{si} (y(t) - \hat{y}(t)) + \hat{f}_{s-1}(t) \\ \vdots \\ \dot{\hat{f}}_2(t) = \Phi_{2i} (y(t) - \hat{y}(t)) + \hat{f}_1(t) \\ \dot{\hat{f}}_1(t) = \Phi_{1i} (y(t) - \hat{y}(t)) \end{cases} \quad (8)$$

where $\hat{x}(t) \in \mathbf{R}^n$, $z(t) \in \mathbf{R}^n$ and $\hat{f}(t) \in \mathbf{R}^m$ are respectively the estimated state vector, the state vector of the observer and the estimated unknown input. Φ_i are the integral gains matrices. $N_i, G_i, L_i, H_i, \Phi_i$ and M_2 are the unknown parameters of the local PMIO which we have to design.

The global state estimation is a fuzzy combination of each

local observer outputs. The overall PMIO dynamics will then be a weighted sum of individual linear PMIO as follows:

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^h h_i(\xi(t))(N_i z(t) + G_i u(t) + L_i y(t) + H_i \hat{f}_s(t)) \\ \hat{x}(t) = z(t) + M_2 y(t) \\ \dot{\hat{f}}_s(t) = \sum_{i=1}^h h_i(\xi(t)) \Phi_{si} (y(t) - \hat{y}(t)) + \hat{f}_{s-1}(t) \\ \vdots \\ \dot{\hat{f}}_2(t) = \sum_{i=1}^h h_i(\xi(t)) \Phi_{2i} (y(t) - \hat{y}(t)) + \hat{f}_1(t) \\ \dot{\hat{f}}_1(t) = \sum_{i=1}^h h_i(\xi(t)) \Phi_{1i} (y(t) - \hat{y}(t)) \end{cases} \quad (9)$$

where the weights $h_i(\xi(t))$, ($i = 1, \dots, h$) are the same as the weights functions used in T-S descriptor model (3). $\hat{f}_j(t)$, $j = 1, \dots, s$ are the estimation of the $(s-1)$ first derivatives of the fault $f(t)$.

The state and the fault estimation errors are given by:

$$e(t) = x(t) - \hat{x}(t) \quad (10)$$

$$e_j(t) = f^{(s-j)}(t) - \hat{f}_j(t), j = 1, 2, \dots, s \quad (11)$$

we assume that $f^{(s)} = 0$.

For subsequently it is assumed that the matrix C is constant. Under Assumption A3 [3], there exists nonsingular matrices $M_1 \in \mathbb{R}^{n \times n}$ and $M_2 \in \mathbb{R}^{n \times m}$ such that:

$$M_1 E + M_2 C = I_n \quad (12)$$

The dynamic estimation error is then described by:

$$\dot{e}(t) = \dot{z}(t) - M_1 E \dot{x}(t) \quad (13)$$

Then, the dynamics of the state and the fault estimation errors are given as the following form:

$$\begin{aligned} \dot{e}(t) &= \sum_{i=1}^h h_i(\xi(t)) [N_i e(t) + (M_1 A_i - L_i C - N_i M_1 E) x(t) \\ &+ (M_1 B_i - G_i) u(t) + (M_1 B_i - H_i) f(t) + H_i e_s(t)] \end{aligned} \quad (14)$$

$$\dot{e}_s(t) = - \sum_{i=1}^h h_i(\xi(t)) \Phi_{si} C e(t) + e_{s-1}(t) \quad (15)$$

\vdots

$$\dot{e}_2(t) = - \sum_{i=1}^h h_i(\xi(t)) \Phi_{2i} C e(t) + e_1(t) \quad (16)$$

$$\dot{e}_1(t) = - \sum_{i=1}^h h_i(\xi(t)) \Phi_{1i} C e(t) \quad (17)$$

If the following conditions hold true $\forall i = 1, \dots, h$:

$$M_1 A_i - L_i C - N_i M_1 E = 0 \quad (18a)$$

$$M_1 E + M_2 C = I_n \quad (18b)$$

$$M_1 B_i - G_i = 0 \quad (18c)$$

$$L_i G - N_i T_2 G - T_1 R_i = 0 \quad (18d)$$

$$M_1 B_i - H_i = 0 \quad (18e)$$

then, the estimation error dynamic (14) becomes:

$$\dot{e}(t) = \sum_{i=1}^h h_i(\xi(t)) [N_i e(t) + H_i e_s(t)] \quad (19)$$

or from equation (18), the above equation is equivalent to:

$$\dot{e}(t) = \sum_{i=1}^h h_i(\xi(t)) [(M_1 A_i + K_i C) e(t) + H_i e_s(t)] \quad (20)$$

where

$$K_i = N_i M_2 - L_i \quad (21)$$

The equations (15)-(17) and (20) can be rewritten in the following augmented form:

$$\dot{\bar{e}}(t) = \sum_{i=1}^h h_i(\xi(t)) (\bar{A}_i + \bar{K}_i \bar{C}) \bar{e}(t) \quad (22)$$

where

$$\bar{e}(t) = \begin{bmatrix} e(t) \\ e_s(t) \\ \vdots \\ e_1(t) \end{bmatrix}, \bar{A}_i = \begin{bmatrix} M_1 A_i & H_i & 0 & \cdots & 0 & 0 \\ 0 & 0 & I_s & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & I_s & 0 \\ 0 & 0 & 0 & \cdots & 0 & I_s \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\bar{K}_i = \begin{bmatrix} K_i \\ -\Phi_{si} \\ \vdots \\ -\Phi_{2i} \\ -\Phi_{1i} \end{bmatrix}, \bar{C} = [C \quad 0 \quad \cdots \quad \cdots \quad 0 \quad 0]$$

The system dynamics $[\bar{A}_i + \bar{K}_i \bar{C}]$ can be stabilized by selecting the gain \bar{K}_i thanks to the detectability of each pair $(\bar{A}_i; \bar{C})$, $\forall i = 1, \dots, h$.

In the sequel and before considering the stability of the estimation error dynamics (22), it is shown how to find matrices M_1 and M_2 such that constraint (18b) is satisfied. For that, rewrite (18b) as follows:

$$[M_1 \quad M_2] \begin{bmatrix} E \\ C \end{bmatrix} = [I_n] \quad (23)$$

A solution $[M_1 \quad M_2]$ exists if [4] :

$$\text{rank} \begin{bmatrix} E \\ C \end{bmatrix} = n \quad (24)$$

Then, a particular solution of (23) using the pseudo inverse matrix denoted by $(\cdot)^+$ is given by:

$$[M_1 \quad M_2] = \begin{bmatrix} E \\ C \end{bmatrix}^+ \quad (25)$$

The proposed observer give the possibility to estimate a large class of faults, because of its multi-integral structure which may change according to the class of faults. Thereafter, the outputs of this observer will be used in fault tolerant control.

IV. FTC FOR T-S DESCRIPTOR SYSTEMS

By using the PMIO (8) and the proposed active fault tolerant control (5), the objective is to determine the parameters of the used observer and the gains Γ_i in order to minimize the impact of actuator faults on the T-S descriptor model output. The system with the fault $f(t) \in \mathbf{R}^{n_f}$ is described by the following T-S model with measurable premise variables:

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^h h_i(\xi(t)) [A_i x(t) + B_i u_f(t) + B_i f(t)] \\ y(t) = Cx(t) \end{cases} \quad (26)$$

The goal is to design the control law $u_f(t)$ such that the system state $x_f(t)$ converges toward the reference state $x(t)$. In order to prove both the stability of the closed-loop system and the convergence of the state and fault estimation errors and according to the equations (5), (8) and (26), the time derivative of the augmented errors (22) become then:

$$\dot{\bar{e}}_a(t) = \sum_{i=1}^h h_i(\xi(t)) (\tilde{A}_i + \tilde{K}_i \tilde{C}) \bar{e}_a(t) \quad (27)$$

where

$$\bar{e}_a(t) = \begin{bmatrix} e(t) \\ e_s(t) \\ \vdots \\ e_l(t) \end{bmatrix}, \tilde{A}_i = \begin{bmatrix} M_1 A_i & (M_1 + I_n) B_i & 0 & \cdots & 0 & 0 \\ 0 & 0 & I_s & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & I_s & 0 \\ 0 & 0 & 0 & \cdots & 0 & I_s \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\tilde{K}_i = \begin{bmatrix} K_i & B_i \Gamma_i \\ -\Phi_{si} & 0 \\ \vdots & \vdots \\ -\Phi_{2i} & 0 \\ -\Phi_{1i} & 0 \end{bmatrix}, \tilde{C} = \begin{bmatrix} C & 0 & \cdots & \cdots & 0 & 0 \\ I_n & 0 & \cdots & \cdots & 0 & 0 \end{bmatrix}$$

The synthesis of the gains \tilde{K}_i of the PMI Observer and those of the controller Γ_i are obtained by solving the LMIs given in the following theorem.

Theorem 1: [5] The PMIO (8) is asymptotically stable, if there exist a symmetrical and definite positive matrix Q and a matrices $\tilde{W}_i = Q\tilde{K}_i$ checking the following LMIs

$$\tilde{A}_i^T Q + Q\tilde{A}_i + \tilde{C}^T \tilde{W}_i^T + \tilde{W}_i \tilde{C} < 0, \forall i \in \{1, \dots, h\} \quad (28)$$

V. SIMULATION EXAMPLE

To illustrate the performances of the proposed approach, let us consider the following nonlinear dynamic system:

$$\begin{aligned} x_1(t) &= x_2(t) \\ x_2(t) &= -2x_1(t) - 3x_2(t) + x_4(t) - x_4^3(t) + 2u(t) \\ 0 &= x_1(t) + x_2(t) - 2x_3(t) \\ 0 &= -x_1(t) - x_2(t) + x_4^3(t) - 5x_4(t) \\ y_1(t) &= x_1(t) + x_3(t) \\ y_2(t) &= x_2(t) + x_4(t) \\ y_3(t) &= x_4(t) \end{aligned} \quad (29)$$

To carry out the proposed PMI observer design, system (29) is rewritten as:

$$\begin{aligned} E\dot{x}(t) &= A(x(t))x(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (30)$$

where

$$A(x(t)) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & -3 & 0 & (1-x_4^2(t)) \\ 1 & 1 & -2 & 0 \\ -1 & -1 & 0 & (x_4^2(t)-5) \end{bmatrix}, B = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

System (30) can be exactly represented by the following fuzzy model:

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^4 h_i(\xi(t)) [A_i x(t) + B_i u(t)] \\ y(t) = Cx(t) \end{cases} \quad (31)$$

$$\text{where } B_i = B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & -3 & 0 & 1 \\ 1 & 1 & -2 & 0 \\ -1 & -1 & 0 & -5 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & -3 & 0 & 1 \\ 1 & 1 & -2 & 0 \\ -1 & -1 & 0 & -1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & -3 & 0 & -3 \\ 1 & 1 & -2 & 0 \\ -1 & -1 & 0 & -5 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & -3 & 0 & -3 \\ 1 & 1 & -2 & 0 \\ -1 & -1 & 0 & -1 \end{bmatrix}$$

The membership functions are given by:

$$\begin{aligned} h_1(x(t)) &= \frac{((1-x_4^2(t))-1)((x_4^2(t)-5)+5)}{-16}, \\ h_2(x(t)) &= \frac{((1-x_4^2(t))-1)(1+(x_4^2(t)-5))}{16}, \\ h_3(x(t)) &= \frac{(3+(1-x_4^2(t)))(x_4^2(t)-5)+5}{16}, \\ h_4(x(t)) &= \frac{(3+(1-x_4^2(t)))(1+(x_4^2(t)-5))}{-16} \end{aligned}$$

Now, following the design of PMIO and the FTC controller algorithm in the above sections, we consider that the matrix $B_i = B$ and the fuzzy model (31) is affected by a varying actuator fault as follows:

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^4 h_i(\xi(t)) [A_i x(t) + B(u(t) + f(t))] \\ y(t) = Cx(t) \end{cases} \quad (32)$$

$u(t) = 10\sin(0, 2\pi t)$ and $f(t) = (0.8(t-20) - 0.08(t-20)^2)\varepsilon(t)$

where

$$\begin{cases} \varepsilon(t) = 1 & \text{for } 10 \leq t \leq 18 \\ \varepsilon(t) = 0 & \text{elsewhere} \end{cases}$$

A. state and fault estimation

According to the given procedure, we design the PMIO based on theorem 1 via the Matlab LMI toolbox. Then we obtain the proportional gains matrices K_i and the integral gains matrices Φ_i for $i = 1, \dots, h$. The state estimation and the fault estimation given by the proposed PMIO are shown on the following figures.

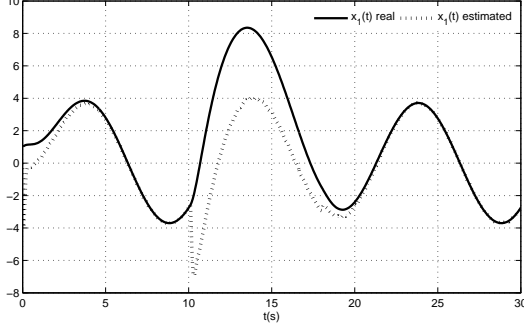


Fig. 1. Nonlinear real states $x_1(t)$ and its estimated $\hat{x}_1(t)$

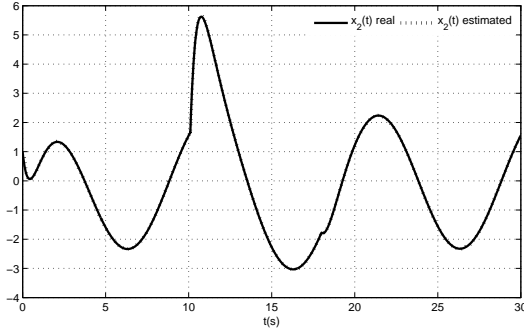


Fig. 2. Nonlinear real states $x_2(t)$ and its estimated $\hat{x}_2(t)$

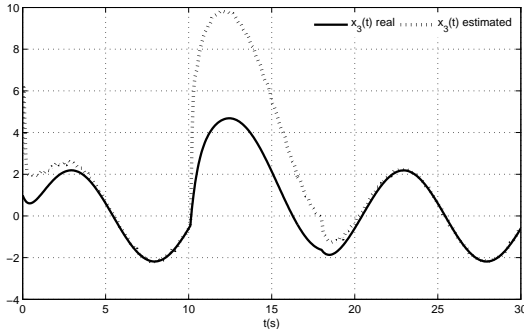


Fig. 3. Nonlinear real states $x_3(t)$ and its estimated $\hat{x}_3(t)$

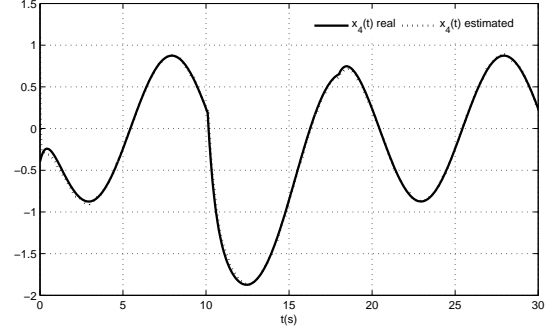


Fig. 4. Nonlinear real states $x_4(t)$ and its estimated $\hat{x}_4(t)$

The behavior of the PMIO is shown in the previous figures (1) to (4). It is observed that the proposed PMIO rebuilds the state by using the estimate of the fault presented in the following figure.

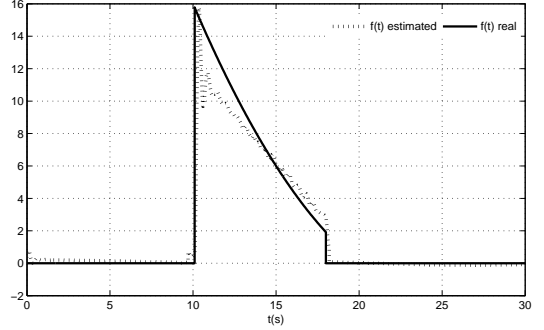


Fig. 5. Actuator fault $f(t)$ and its estimated $\hat{f}(t)$

The estimation of the actuator fault figure (5) is used to design the fault tolerant control.

B. Fault tolerant control of T-S descriptor systems

The control observer based control law given by the equations (9) is designed by solving the LMI optimization problem defined in the theorem 1. The feedback gains Γ_i are given by:

$$\begin{aligned}\Gamma_1 &= \begin{bmatrix} -0.4163 & -3.7974 & 0.3246 & 2.2841 \end{bmatrix}, \\ \Gamma_2 &= \begin{bmatrix} -0.4163 & -3.7974 & 0.3246 & 2.2841 \end{bmatrix}, \\ \Gamma_3 &= \begin{bmatrix} -0.5789 & -1.0026 & 0.4999 & -1.8506 \end{bmatrix}, \\ \Gamma_4 &= \begin{bmatrix} -0.5789 & -1.0026 & 0.4999 & -1.8506 \end{bmatrix}\end{aligned}$$

The following figures illustrate a comparison between the output of the reference model (without fault), the output of the faulty system without FTC and finally the output with FTC. The proposed observer is robust with respect to varying actuator additive fault $f(t)$.

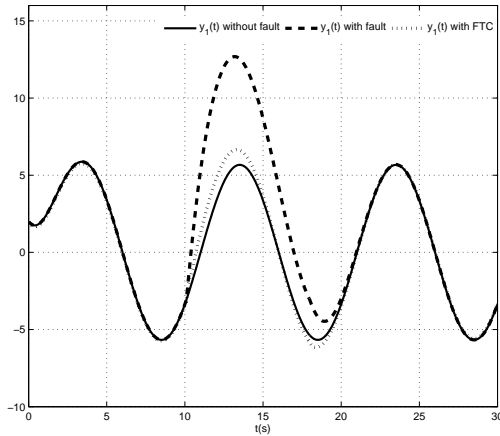


Fig. 6. $y_1(t)$ of the system: without fault, with fault and output with FTC

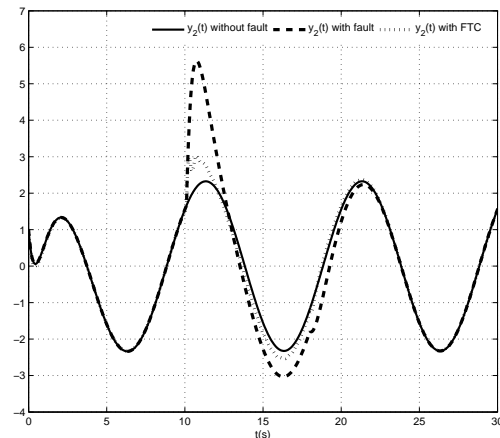


Fig. 7. $y_2(t)$ of the system: without fault, with fault and output with FTC

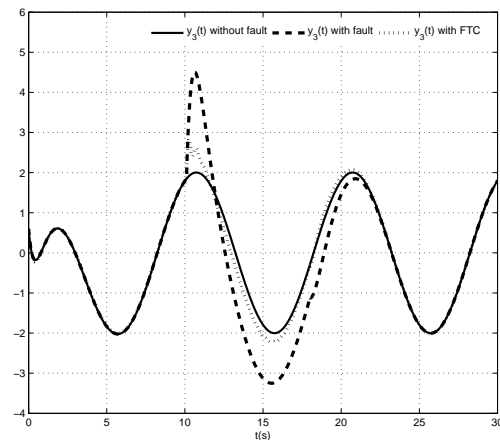


Fig. 8. $y_3(t)$ of the system: without fault, with fault and output with FTC

From these figures, the FTC scheme can accommodate effectively the additive actuator fault. Here, the PMI Observer

shows very good results for the estimation of abrupt actuator fault.

VI. CONCLUSIONS

A combined state, faults estimation and fault tolerant control method have been presented based on PMIO for fuzzy descriptor systems affected by actuator faults. The fault tolerant control requires the simultaneous estimations of the state and faults, obtained by the proposed fuzzy PIMO. This observer admits a greatest potential to estimate time varying faults with a good accuracy simultaneously with the estimation of the state. Sufficient stability conditions are given in terms of LMI. Results have been illustrated in simulation.

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